ASGER AABOE

A COMPUTED LIST OF NEW MOONS FOR 319 B.C. TO 316 B.C. FROM BABYLON: B.M. 40094

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Synopsis

B. M. 40094 is a computed ephemeris from Babylon giving the moment of conjunction of sun and moon for each month from S. E.–8, XII to S. E.–5, XII (i. e., –318, Mar. 31 f.). It is the earliest lunar text belonging to System A that has come to light so far. Though otherwise in strict agreement with the procedures of the later texts of this system, this text is unique in incorporating the function Λ and three new related functions, Y, $\tilde{C}'$, and $\tilde{K}$. These new functions make it possible to solve several problems in the history of Babylonian lunar theory, particularly those concerning relations between mean values of Φ, G, and Λ.

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Introduction

There are two final goals of Babylonian lunar theories: to foretell eclipses, and to predict the duration of various visibility phenomena near new and full moons, most importantly the time from sunset to moonset on the evening of first visibility of the new moon.

In the solution of either kind of problem the determination of the moment of syzygy, i. e., when the moon is either in conjunction or opposition to the sun, is of obvious significance. The fluctuations of the time interval between consecutive syzygies of the same kind are caused by the variation of the solar velocity, with the year as its period, conjointly with the variation of the lunar velocity, with the anomalistic month as its period. In the present paper[1] I shall be concerned with the lunar theory according to System A, in the terminology of ACT,[2] and here the combined effect of the two variable velocities is separated into two periodic additive terms, G and J, so that the time from syzygy to syzygy is

$$29^{\mathrm{d}} + \mathrm{G}^{\mathrm{H}} + \mathrm{J}^{\mathrm{H}},^{3}$$

where G's period is the anomalistic month and J depends on solar longitude.

G belongs to a family of periodic functions (Φ,F,G,Λ,X) from lunar System A, each of them with the anomalistic month as its period. Φ is a pure zig-zag function which runs uninterruptedly, as experience has shown so far, through the entire corpus of lunar System A texts. G is derived from Φ by a set of arithmetical rules of transformation which has long been under control, though it lacked astronomical justification; the meaning of Φ was

[1] My visit to the British Museum, as well as part of my subsequent work, was supported by a grant from the National Science Foundation.

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[2] ACT = O. Neugebauer, *Astronomical Cuneiform Texts*. 3 vols., London, 1955. The reader is referred to this work for all details of theories, methods, and parameters.

[3] $1^{\mathrm{d}} = 6^{\mathrm{H}}$ (large hours) = 6,0 (time) degrees. The large hour is introduced for convenience in modern textual editions.

not at all clear, though it was known that Φ is in precise phase with lunar velocity as represented by Column F.

In a recent article[4] I published some late-Babylonian texts which, in conjunction with a previously published text,[5] threw new light on this family of functions. X was found here for the first time; Λ was already known from two procedure texts in ACT which had taught us how to derive Λ from Φ, but nothing more, and the remaining three functions, Φ, F, and G, are in evidence in all the lunar ephemerides. These texts made it possible to identify all of the members of this family and to give astronomical sense to the established arithmetical rules for converting values of Φ into corresponding values of F, G, and Λ. We can now say that the values of these functions, associated with a given syzygy, have the following significance, beginning with the two that were identified already by KUGLER:[6]

daily progress of moon	$= \mathrm{F}^{\circ}$
length of preceding month	$= 29^{d} + \mathrm{G}^{H}$
length of subsequent 223 months	$= 6585^{d} + \Phi^{H}$
length of preceding 12 months	$= 354^{d} + \Lambda^{H}$
difference between a constant year and the length of preceding 12 months	$= X^{d}$

All of these functions, save perhaps F, are artificial in the sense that they are not directly observable. They represent preliminary values, expressing only the effect of a variable lunar velocity. Indeed, they are not even necessarily correct in the mean, for G and, as we learn from the text published here, also Λ receive corrections for solar anomaly, J and Y, respectively, neither of which has zero as its mean value.

There were two pieces of information in these texts that were crucial in making these identifications possible. The first was a pair of relations between differences in Φ and in G and Λ, respectively, or more precisely, letting Φ_n mean the value of Φ associated with syzygy number n in a certain sequence of syzygies of the same kind, and analogously for the other functions,

$$\Phi_n - \Phi_{n-1} = \mathrm{G}_{n+223} - \mathrm{G}_n \tag{1}$$

and

$$\Phi_n - \Phi_{n-12} = \Lambda_{n+223} - \Lambda_n. \tag{2}$$

[4] ASGER AABOE, *Some Lunar Auxiliary Tables and Related Texts from the Late Babylonian Period.* Mat. Fys. Medd. Dan. Vid. Selsk. **36**, no. 12 (1968).

[5] O. NEUGEBAUER, *"Saros" and Lunar Velocity in Babylonian Astronomy.* Mat. Fys. Medd. Dan. Vid. Selsk. **31**, no. 4 (1957). I shall refer to it as the Saros paper.

[6] F. X. KUGLER, *Die Babylonische Mondrechnung.* Freiburg im Breisgau, 1900.

The second was that when values of Φ were in active use, as in (1) and (2), then Φ was no longer a strict zig-zag function, but it was truncated at effective extrema ($2;13,20^{\mathrm{H}}$ and $1;58,31,6,40^{\mathrm{H}}$, to be exact). Incidentally, it turned out that F was treated similarly; its effective extrema are $15^{\circ/\mathrm{d}}$ and $11;15^{\circ/\mathrm{d}}$.

That relation (1) implies that Φ_n measures, but for a constant, the variable length of the 223 months (the "Saros")[7] following upon syzygy number n may be seen as follows:

If corresponding to syzygy number n the length of the subsequent Saros is called σ_n and that of the preceding lunation or month m_n, we have

$$\sigma_n = \sum_{i=n+1}^{n+223} m_i,$$

hence

$$\sigma_n - \sigma_{n-1} = m_{n+223} - m_n.$$

If we ignore all effects but that of lunar anomaly, i. e., assume that J is constant, or that syzygies are evenly spaced in longitude, we have:

$$m_{n+223} - m_n = \mathrm{G}_{n+223} - \mathrm{G}_n$$

so we obtain, using (1),

$$\sigma_n - \sigma_{n-1} = \Phi_n - \Phi_{n-1}$$

or

$$\sigma - \Phi = \text{constant}.$$

The size of Φ makes it plausible that the value of the constant is 6585^{d}.

Once Φ is identified, a similar argument shows that the relation (2) implies that Λ_n measures, but for a constant, the variable length of the 12 months (the "year") preceding syzygy number n. Let y_n be this year; we then have:

$$y_n = \sum_{i=n-11}^{n} m_i.$$

With notations as above, once again assuming a constant J, and using (2), we get:

[7] I shall use "Saros" to mean an interval of 223 synodic months; the importance of this time interval is that 223 synodic months are very nearly of the same length as 239 anomalistic months. In the texts, "18 years" is used as a technical term for 223 months (actually, 223 months exceed 18^{y} by some 10^{d}).

For a history of the use of "Saros" see O. Neugebauer, *The Exact Sciences in Antiquity*. 2nd edition. Providence, 1957, p. 141 ff.

$$\sigma_n - \sigma_{n-12} = \sum_{i=n+1}^{n+223} m_i - \sum_{i=n-11}^{n+211} m_i$$
$$= \sum_{i=n+212}^{n+223} m_i - \sum_{i=n-11}^{n} m_i$$
$$= y_{n+223} - y_n.$$

Since

$$\sigma_n - \sigma_{n-12} = \Phi_n - \Phi_{n-12}$$

we obtain, using (2),

$$\Lambda_{n+223} - \Lambda_n = y_{n+223} - y_n$$

or,

$$y - \Lambda = \text{constant.}$$

For the last step of the argument I used, strictly speaking, that 223 is relatively prime to 6247, the number period of Λ (and Φ).

The size of Λ makes it plausible that the value of the constant is 354^{d}.

A direct relation between G and Λ will be of importance in the following. We have:

$$y_n - y_{n-1} = \sum_{i=n-11}^{n} m_i - \sum_{i=n-12}^{n-1} m_i$$
$$= m_n - m_{n-12}.$$

Thus, arguing as before,

$$\Lambda_n - \Lambda_{n-1} = \mathrm{G}_n - \mathrm{G}_{n-12}. \tag{3}$$

Using the relations (1) and (2), and the truncated version of Φ, it is now possible to derive schemes for transforming Φ into G and Λ, if one provides an initial value for each. For details I refer to my previous publication; the resulting Φ – G table is, but for a few values near G's maximum, the one given in ACT.[8] The analogous Φ – Λ table appears as Table 3 below.

So far I have summarised, in slightly altered form, the relevant results of my previous article. However, there were then several questions that I had to leave unanswered. The most obvious one was about the relation between G and Λ—not their differences, for that is settled by (3)—but their actual values. One would expect, that

$$\sum_{i=n-11}^{n} (29^{\mathrm{d}} + \mathrm{G}_i^{\mathrm{H}})$$

would be precisely $354^{\mathrm{d}} + \Lambda_n^{\mathrm{H}}$, but that is not so. Thus the connexion between the initial values (or mean values) of G and Λ is not the obvious one, and

[8] ACT I, p. 60.

it was only when the text published here came under control that the proper relation became clear.

It appeared, as hinted above, that even as G receives a correction J for solar anomaly, so also should one apply a correction Y, as I call it, to Λ. With these corrections one has, indeed,

$$\sum_{i=n-11}^{n} (29^{d} + G_i^{H} + J_i^{H}) = 354^{d} + \Lambda_n^{H} + Y_n^{H}, \tag{4}$$

or, at least, very nearly (the small deviations may be explained, in part, by the adjustments of G near its maximum).

The relation (4) implies, that

$$\Lambda_n - \Lambda_{n-1} + Y_n - Y_{n-1} = G_n - G_{n-12} + J_n - J_{n-12}$$

which, together with (3), yields

$$Y_n - Y_{n-1} = J_n - J_{n-12}. \tag{5}$$

The relation (5) is satisfied exactly; further, we learn from our text that Y, as J, is zero on the arc of the ecliptic where the monthly progress of the sun is high ($30^{\circ/\mathrm{m}}$). I shall proceed to show in detail how the decision that Y vanish on the fast arc combined with relation (5) determines Y completely, if J is known.

At the base of J is the solar model of System A. The generating function (which here may be interpreted as the solar velocity in degrees per synodic month) is

$$♓\ 27^{\circ} \text{ to } ♍\ 13^{\circ}:\ w = 28;7,30^{\circ}$$

$$♍\ 13^{\circ} \text{ to } ♓\ 27^{\circ}:\ W = 30^{\circ}.$$

The monthly solar progress in longitude is then either W or w if the sun during the month remains entirely within the fast or the slow arc, respectively. If the place of discontinuity from high to low velocity must be crossed, the standard interpolation rule of System A is employed, i. e., if the distance p of the initial longitude from ♓ 27° is less than $W = 30°$, then the monthly progress $\Delta\lambda$ of the sun is

$$\Delta\lambda = p + q$$

where

$$\frac{p}{W} + \frac{q}{w} = 1,$$

and symmetrically for the other discontinuity. The period of this model is

$$P = \frac{46,23}{3,45} = 12;22,8$$

with

$$\Pi = 46,23 \text{ and } Z = 3,45,$$

i. e., the year has here the canonical value of 12;22,8 synodic months, or, in whole numbers, 46,23 months correspond to 3,45 revolutions in the ecliptic, i. e., 3,45 years.

This solar model can advantageously be reduced to a distribution of 46,23 intervals on the ecliptic,[9] 24,15 of length 0;8° on the fast arc, and 22,8 of length 0;7,30° on the slow; the monthly progress of the sun will then always be 3,45 intervals, regardless of their length.

Column J is closely and simply tied to this solar model. If λ_n is the solar longitude at syzygy number n, and the sun during the previous month has travelled the distance s within the slow arc ($0 \leqq s \leqq w$), then

$$\mathrm{J}_n = \mathrm{J}(\lambda_n) = -\frac{s}{w} \cdot 0;57,3,45^{\mathrm{H}}.$$

On most of the fast arc we have that $s = 0$, so $\mathrm{J} = 0$; on most of the slow arc $s = w$, so $\mathrm{J} = -0;57,3,45$. Only when a place of discontinuity of the solar velocity is passed in the course of the previous month, i. e., when λ_n is between ♓ 27° and ♈ 25;7,30° or between ♍ 13° and ♎ 13°, do we get intermediate values of J, and they depend linearly on λ_n (see Figure 1). I shall show later that this J-function makes very good sense and that it can be derived from the solar model and the lunar velocity, but for the moment we shall take it as given.

If we now are to derive the analogous correction, Y, to Λ, we begin with relation (5) which states that the monthly difference in Y is the 12-monthly difference in J or, more precisely,

$$\Delta \mathrm{Y} = \mathrm{Y}_n - \mathrm{Y}_{n-1} = \mathrm{J}_n - \mathrm{J}_{n-12}. \tag{5}$$

To advance 12 months means, in terms of the distribution version of the solar model, an advance in longitude of

$$12 \cdot Z = 12 \cdot 3,45 = 45,0 \equiv -1,23 \text{ intervals } (\mathrm{mod.}\ \Pi)$$

[9] Asger Aaboe, *On Period Relations in Babylonian Astronomy.* Centaurus 1964, vol. 10, pp. 213–231.

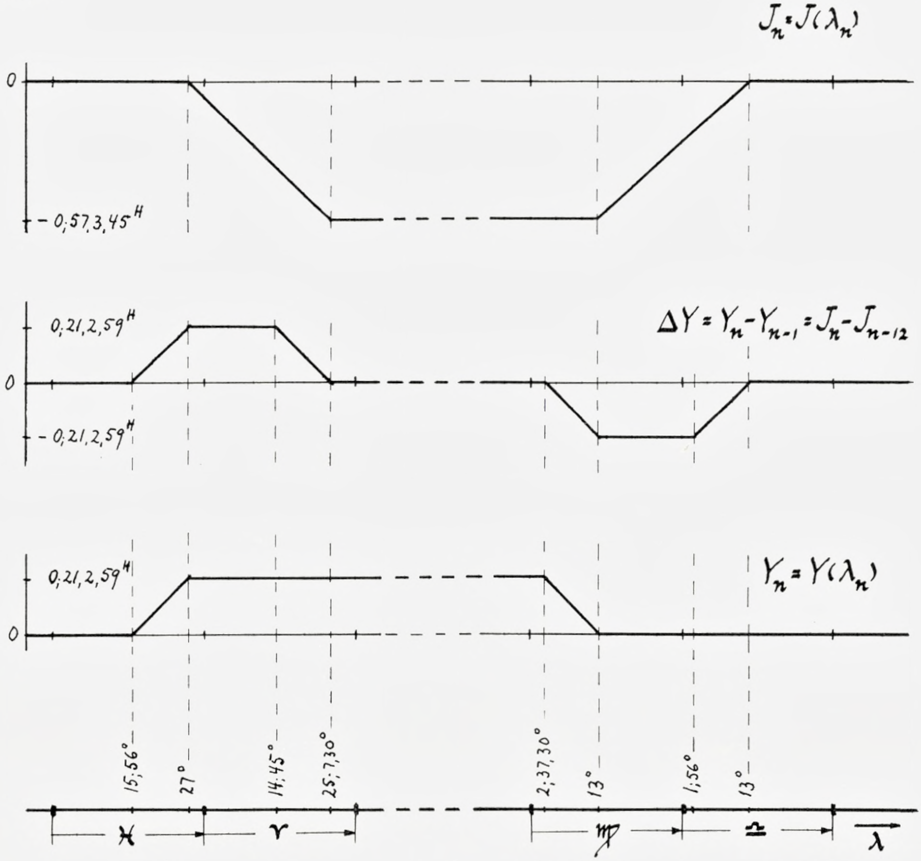

Fig. 1.

or a lag of 1,23 intervals; on the fast arc, 1,23 intervals amount to 11;4°, and on the slow to 10;22,30°. Thus, λ_{n-12} will always be 1,23 intervals ahead of λ_n in the ecliptic.

If λ_n and λ_{n-12} both lie on one of the predominant stretches of the ecliptic where J is constant, ΔY will be zero. If λ_n and λ_{n-12} both lie in one of the two transitional zones for J, both of length 3,45 intervals, ΔY will, but for its sign, be

$$|J_n - J_{n-12}| = \frac{1,23}{3,45} \cdot 0;57,3,45 = 0;21,2,59^{H}.$$

When we are moving into the slow arc, this contribution will be positive, and it is negative in the other transitional zone. When only one of the longitudes

λ_n and λ_{n-12} is in a transitional zone of J, ΔY will assume an intermediary value, and will depend linearly on λ_n, as shown in Figure 1.

If to this we now add the demand that Y itself be zero on the fast arc, or most of it, Y_n is completely determined as a function of λ_n (see again Figure 1). The value of Y will be

$$Y = +0;21,2,59^{\mathrm{H}}$$

on most of the slow arc, but the jumps of the solar model will be preceded by short transitional zones, as is readily seen.

It is convenient and, as we shall see, useful to give Y_n directly as a function of λ_n, avoiding J as an intermediary:

λ_n between ♍ 13° and ♓ 15;56°: $Y_n = 0$
λ_n between ♓ 15;56° and ♓ 27°: $Y_n = 0;1,54,7,30 \cdot (\lambda_n - ♓\ 15;56)$
λ_n between ♓ 27° and ♍ 2;37,30°: $Y_n = 0;21,2,59$
λ_n between ♍ 2;37,30° and ♍ 13°: $Y_n = 0;2,1,44 \cdot (♍\ 13 - \lambda_n)$.

The zones of transition are so short for Y (1,23 intervals compared to the monthly advance of 3,45) that transitional Y-values are avoided more often than not.

I should emphasise that Y_n is not the same as

$$\sum_{i=n-11}^{n} J_i;$$

Y agrees with J only in its differences, but the condition that Y be zero on the fast arc is entirely independent of J.

It is now possible to compute mean values, $\bar{J}$ and $\bar{Y}$, for J and Y. Since the values in one transitional zone complement those in the other, the non-zero values are effectively in play in 22,8 intervals (the number of intervals of the slow zone) out of the entire 46,23. Thus:

$$\bar{J} = -0;57,3,45 \cdot \frac{22,8}{46,23} = -0;27,13,45,\ldots^{\mathrm{H}}$$

and

$$\bar{Y} = 0;21,2,59 \cdot \frac{22,8}{46,23} = 0;10,2,40,\ldots^{\mathrm{H}}$$

Applying these mean corrections I can now get agreement where I failed earlier. In my previous publication I computed

$$\max \sum_{i=1}^{12} G_i \equiv 9;32,21,43,\ldots^{H} \pmod{6^{H}}$$

which did not agree with

$$\max \Lambda = 3;55,33,20^{H},$$

even modulo 6^{H}. However,

$$\begin{aligned}\max \sum_{1}^{12} G + 12\,\bar{J} &= 9;32,21,43,\ldots - 5;26,45,6,\ldots \\ &= 4;5,36,36,\ldots^{H}\end{aligned}$$

and

$$\begin{aligned}\max \Lambda + \bar{Y} &= 3;55,33,20 + 0;10,2,40,\ldots \\ &= 4;5,36,0,\ldots^{H}.\end{aligned}$$

The slight deviation is, in part, due to the adjustment of G near its maximum. As said, it is in general so that

$$\sum_{i=n-11}^{n} (29^{d} + G_i^{H} + J_i^{H}) = 354^{d} + \Lambda_n^{H} + Y_n^{H},$$

at least to several sexagesimal places, so the rôle of Λ, when corrected by Y, can well be to provide a much needed control for the summation of G and J. I shall elaborate on this point below.

Further, I found in my previous paper that the sum of Λ, converted into days, and X was very constant, so

$$354^{d} + \Lambda + X = 6,5;9,33,\ldots^{d}$$

which is near a value of the year, though too small; thus I interpreted X as the variable epact, i. e., the difference between a constant year and the variable length of 12 months. Applying the correction $\bar{Y}$ we now have:

$$354^{d} + \Lambda + \bar{Y} + X = 6,5;11,13,\ldots^{d}$$

which is a better year value, though still too small.

It is now a reasonable guess that even as Λ and G receive corrections for solar anomaly so also does Φ, though there is at present no textual evidence for it. Calling this hypothetical correction S we would then have for the length of the Saros

$$\sigma_n = 6585^{d} + \Phi_n + S_n = \sum_{i=n+1}^{n+223} (29^{d} + G_i + J_i),$$

whence

$$\begin{aligned}\sigma_n - \sigma_{n-1} &= \Phi_n - \Phi_{n-1} + \mathrm{S}_n - \mathrm{S}_{n-1} \\ &= \mathrm{G}_{n+223} - \mathrm{G}_n + \mathrm{J}_{n+223} - \mathrm{J}_n\end{aligned}$$

so

$$\mathrm{S}_n - \mathrm{S}_{n-1} = \mathrm{J}_{n+223} - \mathrm{J}_n .$$

An advance of 223 months corresponds to an advance in longitude of

$$3{,}43.Z = 13{,}56{,}15 \equiv 1{,}21 \text{ intervals } (\mathrm{mod}\ \Pi)$$

(observe, that to go one Saros forward or 12 months back leads to solar positions within 2 intervals of each other). The 1,21 intervals amount to 10;7,30° on the slow arc and 10;48° on the fast.[9a] If the two longitudes λ_n and λ_{n+223} both lie in a transitional zone of J, the corresponding change in J will, but for its sign, be

$$|\mathrm{J}_{n+223} - \mathrm{J}_n| = \frac{1{,}21}{3{,}45} \cdot 0;57{,}3{,}45 = 0;20{,}32{,}33^{\mathrm{H}}.$$

When we move from the fast zone into the slow,

$$\Delta\mathrm{S} = \mathrm{J}_{n+223} - \mathrm{J}_n$$

will be negative.

If we now require that S be zero on most of the fast arc, as are Y and J, an argument completely analogous to that for Y shows, that

$$\mathrm{S} = -0;20{,}32{,}33^{\mathrm{H}}$$

on most of the slow arc. As for Y, there will be transitional zones for S, but they will be of length 1,21 intervals instead of 1,23 intervals.

The effective mean value of S, $\overline{\mathrm{S}}$, will be

$$\overline{\mathrm{S}} = -0;20{,}32{,}33 \cdot \frac{22{,}8}{46{,}23} = -0;9{,}48{,}9, \ldots^{\mathrm{H}} = -0;1{,}38{,}1, \ldots^{\mathrm{d}}.$$

It is very plausible, indeed, that Φ requires a correction of this sort. The mean value of Φ when not truncated is

$$\mu_\Phi = 2;7{,}26{,}23{,}20^{\mathrm{H}} = 0;21{,}14{,}23, \ldots^{\mathrm{d}};$$

but if one considers the effective Φ-function, truncated at 2;13,20 and 1;58,31,6,40, one can readily compute the effective mean value of Φ by finding the "area" under the truncated curve, to use modern terminology.

[9a] Cf. ACT No. 204, Section 7, where line 18 can now be restored.

It is
$$\overline{\Phi} = 2;7,5,20,57,\ldots^{\mathrm{H}} = 0;21,10,53,\ldots^{\mathrm{d}}.$$

If we use the classical value for the mean synodic month of

$$\bar{m}_{\mathrm{B}} = 29;31,50,8,20^{\mathrm{d}}$$

which derives from System B, and which was used by HIPPARCHOS and PTOLEMY, we get for the mean Saros:

$$\bar{\sigma}_{\mathrm{B}} = 3,43 \cdot \bar{m}_{\mathrm{B}} = 1,49,45;19,20,48,20^{\mathrm{d}}.$$

There is no explicit value for the mean synodic month in System A; however, NEUGEBAUER derived one in the Saros paper from the value of the anomalistic month and the period relation of F and Φ, which relates the anomalistic to the synodic month, and he got

$$\bar{m}_{\mathrm{A}} = 29;31,50,19,11,\ldots^{\mathrm{d}}.$$

This value yields a mean Saros of

$$\bar{\sigma}_{\mathrm{A}} = 1,49,45;20,1,17,\ldots^{\mathrm{d}}.$$

For either of these values of the mean Saros, the fractional part is less than both μ_{Φ} and $\overline{\Phi}$. However, if the latter values be corrected by the hypothetical $\overline{\mathrm{S}}$ (in days), we get:

$$1,49,45 + \mu_{\Phi} + \overline{\mathrm{S}} = 1,49,45;19,36,22,\ldots^{\mathrm{d}}$$

and

$$1,49,45 + \overline{\Phi} + \overline{\mathrm{S}} = 1,49,45;19,32,52,\ldots^{\mathrm{d}}.$$

Though no perfect agreement is reached, it still seems reasonable that a correction like S should be applied to Φ.

Further, by brute numerical "integration" of G, NEUGEBAUER found the following effective "area" mean value of G:

$$\overline{\mathrm{G}} = 3;38,15,1,\ldots^{\mathrm{H}} = 0;36,22,30,\ldots^{\mathrm{d}}$$

which, after application of the mean value $\overline{\mathrm{J}}$ in days, implies a value for the mean synodic month of

$$\bar{m} = 29^{\mathrm{d}} + \overline{\mathrm{G}} + \overline{\mathrm{J}} = 29;31,50,12,\ldots^{\mathrm{d}}.$$

Thus one gets the following value of the mean Saros:

$$3,43 \cdot (29^{\mathrm{d}} + \overline{\mathrm{G}} + \overline{\mathrm{J}}) = 1,49,45;19,36,42,\ldots^{\mathrm{d}},$$

which, curiously enough, agrees better with the value derived from μ_{Φ} than with that derived from $\overline{\Phi}$.

All of the values of the mean Saros, and in particular the one derived from the reconstructed System A value of the mean synodic month, are close to

$$\bar{\sigma} = 6585\tfrac{1}{3}{}^{\mathrm{d}}$$

which is assigned to the ancients by PTOLEMY,[10] and which he calls superficial. I cannot help suspecting that the relation

$$223 \text{ months} = 6585^{\mathrm{d}} + \tfrac{1}{3}{}^{\mathrm{d}}$$

somehow played a fundamental rôle in the construction of the more refined schemes, though I still cannot see what it was.

In these discussions of the various corrections for the effect of solar anomaly I have, as I said, taken J as given. However, I shall now show that once the solar model is agreed upon, J is essentially determined by the decision to let it vanish on the fast arc. On the slow arc, syzygies happen

$$30^{\circ} - 28;7,30^{\circ} = 1;52,30^{\circ}$$

sooner in longitude than on the fast arc. If we now use $12;11,27^{\circ/\mathrm{d}}$ as the difference velocity between the moon and the sun, i. e., assume a constant lunar velocity and ignore the relatively slight variation in solar velocity, then syzygies will happen, roughly,

$$\frac{1;52,30^{\circ}}{12;11,27^{\circ/\mathrm{d}}} \cdot 6^{\mathrm{H/d}} = 0;55,22,10,\ldots^{\mathrm{H}}$$

sooner in time on the slow arc than on the fast. If it is then decided that no correction to the time interval between consecutive syzygies is desired when both syzygies happen on the fast arc, a correction of the order of

$$-0;55,22,10^{\mathrm{H}}$$

should be applied when both syzygies occur on the slow arc. Further, it is readily seen that if one of the two consecutive syzygies is in the fast arc, and the other in the slow, then the required correction is found by precisely the same sort of rules that yield transitional J-values.

The correction we find in the texts is, of course, not this, but

$$J = -0;57,3,45^{\mathrm{H}},$$

[10] *Almagest* IV, 2.

yet it is of the right order of magnitude, and, as I just said, the rules for finding transitional J values are what one expects. The difference velocity between moon and sun that yields the actual J is, as a simple computation shows, 11;50, . . . $^{\circ/d}$ which is rather low. I have sought, but in vain, for a derivation of precisely $-0;57,3,45$ which satisfied me, and I have failed to see the particular attractiveness of this number. It is, of course, nicely divisible by 3,45 (the Z of the solar model), but so are many other numbers of the same order of magnitude. That 0;57,3,45 is too large obviously does not matter in the long run, for

$$29^{d} + \bar{G} + \bar{J}$$

is a very good value for the mean synodic month. I believe at present that this choice of a value for J may well be motivated by a desire for the pleasant initial value of G,

$$G = 2;40^{H},$$

but it is clear that the order of magnitude is fixed, so the freedom of choice is quite restricted.

It is now possible to attempt a reconstruction of the theory and methodology underlying the procedures for predicting syzygies.

The basic decision is that the effect of lunar and solar anomaly be separated into independent, additive terms, so:

(i)		1 month	$= 29^{d} + G^{H} + J^{H}$
(ii)	1 Saros =	223 months	$= 6585^{d} + \Phi^{H} + S^{H}$
(iii)		12 months	$= 354^{d} + \Lambda^{H} + Y^{H}$

where G, Φ, and Λ depend on lunar anomaly, and J, S, and Y on solar longitude.

The solar model, and the condition that J vanish on the arc of high monthly solar progress, combine to determine J, and hence $\bar{J}$, as we have just seen. Assuming that G, Φ, and Λ vary independently of J, S, and Y, or can be taken to be constant, we can now derive S and Y from J and the decision that they, too, be zero on the fast arc; thus $\bar{S}$ and $\bar{Y}$ are determined.

Turning now to Φ, we observe first that a value of the mean synodic month and S determine the mean value of Φ. Assuming next that J and S are constant, a theoretical argument shows that Φ is in phase with the lunar velocity F.[11] A value of Φ's amplitude, and the decision that Φ be a truncated

[11] For such an argument cf. loc. cit. in note 4, p. 10. If one wishes to check how successful the Babylonians were in bringing Φ and F into phase with the actual lunar velocity, it is particularly convenient to consider the conjunction which happened at the end of S.E. 80, VIII.

zig-zag function finally determine Φ completely. This last step raises several questions which I cannot answer satisfactorily; I shall return to them below.

As for G, a value of the mean synodic month and $\bar{J}$ determine the mean value of G, $\bar{G}$. Further, (i) and (ii), and the assumption that G and Φ vary independently of J and S, establish the fundamental relation between differences in G and Φ

$$\Phi_n - \Phi_{n-1} = G_{n+223} - G_n$$

which determines G but for an additive constant. Finally, $\bar{G}$ serves to fix that constant.

Λ is treated as G.

It is clear that the remaining questions, except for those concerning arithmetical details of adjusting parameters to pleasant values, are raised by Φ. The central rôle of Φ is obvious, and it is now apparent that Φ was in continuous use—in the strong sense that its values computed month by month connect the earliest to the latest texts—since times already before the System A schemes reached their final form. However, I am still at a loss to explain in a satisfactory manner how the amplitude of Φ can be derived from the sort of observations which were recorded by the Babylonians, nor am I yet quite convinced of the desirability of truncating the zig-zag function which is chosen to represent it. And there is still the uncomfortable fact that Φ is found side by side with early and primitive solar models,[11a] while S as constructed above depends on the fully developed System A solar scheme. We can only hope that the appearance of new texts will help us solve these problems.

Text: B.M. 40094

B.M. 40094 (81–2–1, 59).
Provenance: Babylon (B.M. number).
Contents: K, M, Λ, Y, Č', K for new moons, month by month, for Philip Arrhidaeus 4, XII to 7, XII (= S.E.–8, XII to –5, XII).
Transcription: Table 2, complemented by Table 1.
Photograph: see Plate.

Here Φ_1 assumes precisely the value m_Φ, so one would expect the moon to be near its apogee. Dr. John Britton drew my attention to the fact that there happens to be a solar eclipse at this conjunction (–231 Nov. 19, 7;44^h a.m. G.M.T.), so the desired information is readily available. It turns out that the moon is only about $1\frac{1}{2}°$ from its apogee at the moment of conjunction. Since their period relation is good, it is clear that F and Φ were very well in phase with the actual lunar velocity throughout the relevant period.

[11a] Cf. A. Aaboe and A. Sachs, *Two Lunar Texts of the Achaemenid Period from Babylon.* To appear in Centaurus, 1969.

Description of Text

The text is a fragment belonging, as its curvature shows, to the right half of what was probably a very wide tablet of the shape characteristic of lunar ephemerides. Top and bottom, but no other, edges are preserved; the obverse has 20 lines, the reverse 18. The surface is crumbling rather badly. I am convinced that the text is a copy from a poorly preserved exemplar, for there is an unusual number of isolated errors of the sort readily committed in copying a bad text (e. g., 8 for 5, and 5 for 8).

Columns III and IV are run into each other, as are Columns V and VI. The scribe's hand is such that it is often difficult to distinguish between his "tab", "20", and ".", where "." denotes the separation mark consisting of two diagonal wedges, used for zero.

Critical Apparatus

Obv. I, 4. [1,4]0,4*8*: should be 1,40,45.
Obv. II, 6. 2,15,5: should be 2,17,5.
Obv. II, 8. 1,49,*33*: should be 1,49,*43*.
Obv. II, 9. 3,48,*15*: should be 3,48,*25*.
Rev. II, 8. 4],57,4*4*: should be 4,57,4*6*.
Rev. II,14. 1,*4*9,*5*: should be 1,*39*,*35*.
Obv. III, 1. 3,*5*,10: copyist's error for 3,*8*,10; "tab" is followed by what may be "*šá* m[u]" (for the year) although the "*šá*" could be read "4".
Obv. III, 2. 3,4*0*,45: should be 3,4*4*,45.
Obv. III, 9. 1*8*,29,16,42,46,40: should be 17,29,16,42,46,40.
Obv. III,10. 15,15: should be 1*8*,15.
Obv. III,11. 6,27,*54*,*53*,*20*: should be 6,27,*57*,*46*,*40*.
Obv. III,15. 3,10,*4*4,15,33,20: should be 3,10,*5*4,15,33,20.
Obv. III,19. 3,3*4*,12,2,13,20: should be 3,3*1*,12,2,13,20.
Rev. III, 7. 1,33,25,53,5*8*, . . .: should be 1,33,25,53,5*6*,6,40.
Rev. III, 8. *1*,23,36,45,47,13,20: should be *2*,23,36,45,47,13,20.
Rev. III,13. 3,29,11,*6*,40: should be 3,29,11,40; a very natural slip of the stylus for a scribe accustomed to the frequently occurring endings of nice numbers.
Rev. III,18. Λ-value should be denoted lal.
Obv. IV,13. Value should be 21,*2*,59, but traces in the second place look like ⌞3⌟ rather than ⌞2⌟.
Col. IV. Except for the first four lines, the values are denoted lal instead of tab, perhaps in imitation of Col. J.

Table 1.

	$[T_1]$	$[\Phi_1]$	$[B_1]$	$[C_1]$	$[G_1]$	$[J_1]$	$[C'_1]$
Obv.	S.E. −8, XII	2, 7,53, 8,53,20	♈ 11;26,15	3, 0,57,30	3,42,20	− 29,17,31,30	− 9,40,45
	−7, I	2,10,39, 4,26,40	♉ 9;33,45	3,19,42,30	3,16,31,21,28,53,20	− 57, 3,45	− 9,22,30
	II	2,13,25	♊ 7;41,15	3,31, 4,30	2,50,48,20	− 57, 3,45	− 5,41
	III	2,16,10,55,33,20	♋ 5;48,45	3,35,26,30	2,40	− 57, 3,45	− 2,11
	IV	2,15,12,46,40	♌ 3;56,15	3,32,48,30	2,40	− 57, 3,45	+ 1,19
	V	2,12,26,51, 6,40	♍ 2; 3,45	3,23,10,30	2,41,46, 6,40	− 57, 3,45	+ 4,49
	VI	2, 9,40,55,33,20	♎ 1;20	3, 5,46,40	3, 2,31,21,28,53,20	− 22,11,27,30	+ 8,41,55
	VII	2, 6,55	♏ 1;20	2,45,46,40	3,28,20		+ 10
	VIII	2, 4, 9, 4,26,40	♐ 1;20	2,31,28	3,54, 8,38,31, 6,40		+ 7, 9,20
	IX	2, 1,23, 8,53,20	♑ 1;20	2,25, 9,20	4,19,57,17, 2,13,20		+ 3, 9,20
	X	1,58,37,13,20	♒ 1;20	2,26,50,40	4,45,45,55,33,20		− 50,40
	XI	1,59,44,37,46,40	♓ 1;20	2,36,32	4,55, 2,57,46,40		− 4,50,40
	XII	2, 2,30,33,20	♈ 1; 3,45	2,54, 2,30	4,32,30,51,51, 6,40	− 8,14,32,30	− 8,45,15
	XII$_2$	2, 5,16,28,53,20	♈ 29;11,15	3,12,47,30	4, 6,42,13,20	− 57, 3,45	− 9,22,30
	−6, I	2, 8, 2,24,26,40	♉ 27;18,45	3,26,55,30	3,40,53,34,48,53,20	− 57, 3,45	− 7, 4
	II	2,10,48,20	♊ 25;26,15	3,34, 3,30	3,15, 4,56,17,46,40	− 57, 3,45	− 3,34
	III	2,13,34,15,33,20	♋ 23;33,45	3,34,11,30	2,49,33,20	− 57, 3,45	− 4
	IV	2,16,20,11, 6,40	♌ 21;41,15	3,27,19,30	2,40	− 57, 3,45	+ 3,26
	V	2,15, 3,31, 6,40	♍ 20;16	3,13, 9,20	2,40	− 43,14,26,30	+ 7, 5, 5
	VI	2,12,17,35,33,20	♎ 20;16	2,53, 9,20	2,42,22,57,46,40		+ 10
Rev.	VII	2, 9,31,40	♏ 20;16	2,35,53,36	3, 3,57,46,40		+ 8,37,52
	VIII	2, 6,45,44,26,40	♐ 20;16	2,26,37,52	3,29,46,25,11, 6,40		+ 4,37,52
	IX	2, 3,59,48,53,20	♑ 20;16	2,25,22, 8	3,55,35, 3,42,13,20		+ 37,52
	X	2, 1,13,53,20	♒ 20;16	2,32, 6,24	4,21,23,42,13,20		− 3,22, 8
	XI	1,58,27,57,46,40	♓ 20;16	2,46,50,40	4,47, 9,11,51, 6,40		− 7,22, 8
	XII	1,59,53,53,20	♈ 18;48,45	3, 5,52,30	4,54,18,16,17,46,40	− 44,15,18,30	− 9,30,55
	−5, I	2, 2,39,48,53,20	♉ 16;56,15	3,22,46,30	4,31, 4,26,40	− 57, 3,45	− 8,27
	II	2, 5,25,44,26,40	♊ 15; 3,45	3,32,40,30	4, 5,15,48, 8,53,20	− 57, 3,45	− 4,57
	III	2, 8,11,40	♋ 13;11,15	3,35,34,30	3,39,27, 9,37,46,40	− 57, 3,45	− 1,27
	IV	2,10,57,35,33,20	♌ 11;18,45	3,31,28,30	3,13,38,31, 6,40	− 57, 3,45	+ 2, 3
	V	2,13,43,31, 6,40	♍ 9;26,15	3,20,13,30	2,48,19,15,33,20	− 57, 3,45	+ 5,37,30
	VI	2,16,29,26,40	♎ 9;12	3, 0,32	2,40	− 7,13,40,30	+ 9,50,45
	VII	2,14,54,15,33,20	♏ 9;12	2,40,32	2,40		+ 10
	VIII	2,12, 8,20	♐ 9;12	2,28,19,12	2,43, 0,33,20		+ 6, 6,24
	IX	2, 9,22,24,26,40	♑ 9;12	2,24, 6,24	3, 5,24,11,51, 6,40		+ 2, 6,24
	X	2, 6,36,28,53,20	♒ 9;12	2,27,53,36	3,31,12,50,22,13,20		− 1,53,36
	XI	2, 3,50,33,20	♓ 9;12	2,39,40,48	3,57, 1,28,53,20		− 5,53,36
	XII	2, 1, 4,37,46,40	♈ 8;26,15	2,58,57,30	4,22,50, 7,24,26,40	− 23,12,19,30	− 9,38,21

Rev. line 5. Col. IV,5 is empty; the Y-value should be 8,14,32,30 tab. In Col. V,5 the text has the value 8,12,35,30 tab; this should be 3,35,55 tab. In Col. VI,5 we read 3,36,45 with the final 5 damaged; this should be 0;7,29 tab. I believe that the scribe copied from an exemplar which, like his copy, occasionally

TABLE 2.

	I (K)	II (M)		III (Λ)	IV (Y)	V (C̃')	VI (K̃)
Obv.	[3,3],22	29	5,29,46 šú	3,5,10 tab šá(?) m[u](?)	21,2,59 tab	3,2[7,30 tab]	[3,32,40 tab]
	[2,10].5	29	3,19,40 šú	3,40,45	21,2,59 tab	2,8 tab	[4,7,56 tab]
	[1]48,3	28	1,31,38 šú	3,55,33,20	21,2,59 tab	1,. t[ab]	[4,17,36 tab]
	[1,4]0,48	29	5,50,53 šú	3,54,15	21,2,59 tab	8 lal	[4,15,10 tab]
	⌞1,44,15⌟	29	4,6,37 šú	3,33,8,53,20		1,[19,30 lal]	[3,31,49 tab]
	[1,49,31]	⌞28⌟	2,15,5 šú	2,48,1,52,16,6,40		2,24 lal	[2,45,38 tab]
	[2,49,2]	[ ]	5,28,3 šú	1,57,51,.,25		3,41,[20 lal]	[1,54,10 tab]
	[3,38,20]	[ ]	1,49,33 šú	1,7,40,.8,33,53,20		3,22,[8 lal]	[1,4,18 tab]
	[4,1,18]	[ ]	3,48,15 šú	18,29,16,42,46,40		1,53,4[0 lal]	[15,35 tab]
	[4,23,6]	[	5,2]5,19 šú	15,15 lal		25,4 [lal]	[18,40 lal]
	[4,44,55]	[	40,2]4 šú	6,27,54,53,20 lal		1,3,28 [tab]	[5,23 lal]
	[4,50,12]	[	1,50],12 šú	40,27,1,3,53,20 tab		2,32 tab	[42,59 tab]
	[4,15,31]	[	3,3]4,41 šú	1,30,37,52,55 tab	21,///,59 lal	3,27,30 tab	[1,55,8 tab]
	[3,0,15]	[	3]4,26 šú	2,20,48,44,46,6,40	21,2,59 lal	3[27,30 tab]	[2,45,19 tab]
	[2,36,47]	[	3,5]7,39 šú	3,10,44,15,33,20	21,2,59 lal	2,4,3[0 tab]	[3]34,[1 tab]
	[2,14,27]	[	1,4]3,12 šú	3,46	21,2,59 lal	[41,30 tab]	4,[7,44 tab]
	[1,52,25]	[	5,50,4]6 šú	3,55,33,20	21,2,59 lal	[41,30 lal]	4[15,55 tab]
	[1,46,22]	[	4,4,2]4 šú	3,53,47,13,20	21,2,59 lal	2,[4,30 lal]	[4,12,45 tab]
	[2,3,50]	[	2,0,3]4 šú	3,34,12,2,13,20		3,4[1,20 lal]	[3,27,31 tab]
	[2,52,23]	[	5,8,11] šú	2,45,13,51,15		[3,41,20 lal]	[2,41,32 tab]
Rev.	[3,12,36]	[	1,55,36 šú]	[1,55,2,5]9,23,53,20		[2,12,48 lal]	[1,52,50 tab]
	[3,34,24]	[	4,21,12 šú]	1,4,52,7,32,46,40		44,16 lal	[1,4,8 tab]
	[3,56,13]	[	24,59 šú]	14,41,15,41,40 tab		44,16 tab	15,26 [tab]
	[4,18,1]	[	2,6,57 šú]	19,1,17,46,40 lal		2,12,48 tab	16,48 l[al]
	[4,39,47]	[	3,27,10 šú]	4,21,0,52,46,40 lal	<8,14,32,30>	8,12,35,30 tab	3,36,55 [///]
	[4,0,32]	[	5,26,37] šú	43,15,2,5 tab	21,2,59 lal	3,27,30 tab	1,7,4[6 tab]
	[3,25,34]	[	2,1],2 šú	1,33,25,53,58,. . . .	21,2,59 lal	2,4,30 tab	1,56,[33 tab]
	[3,3,15]	[	4]57,44 šú	1,23,36,45,47,13,20 tab	21,2,59 lal	41,30 tab	2,4[5,20 tab]
	[2,40,56]	[	] 2,16,50 šú	3,13,31,40 tab	21,2,59 lal	41,30 lal	3,3[3,53 tab]
	[2,18,38]	[ ]	5,58,12 šú	3,47,14,4,26,40 tab	21,2,59 lal	2,4,30 lal	4,[6,12 tab]
	[1,56,53]	[28]	4,1,19 šú	3,55,33,20 tab	7,13,40,30 lal	3,32,5 lal	[3,59,15 tab]
	[2,42,37]	[2]8	1,18,42 šú	3,53,10,22,13,20 tab		3,41,20 lal	[3,49,29 tab]
	[2,50]	[2]8	4,28,42 šú	3,29,11,6,40 tab		2,19,12 lal	[3,26,52 tab]
	[2,49,7]	[2]8	1,49,5 šú	2,42,25,50,13,53,20 tab		50,40 lal	[2,41,35 tab]
	[3,7,30]	[2]8	4,32,4 šú	1,52,14,58,22,46,40 tab		37,52 tab	[1,52,52 tab]
	[3,29,19]	[ ]	1,2,46 šú	1,2,4,6,31,40 tab		3,6,24 t[ab]	[1,4,10 tab]
	[3,51,8]	[2]9	3,11,36 šú	11,56,23,33,53,20		3,34,5[6 tab]	[15,31 tab]
	[3,49,59]	[2]9	5,21,38 šú	19,29,26,40 <lal>	21,2,59 lal	3,27,[30 tab]	[5,1 tab]

B.M. 40094 (81-2-1,59)

ran the columns together and in which line 5 of the reverse was damaged; and that he copied what he saw in the correct line, but shifted one column to the right.

Obv. V.	All readings are very uncertain.
Obv. V, 3.	1, . ta[b]: reading not certain, might even be 1 u[š].
Rev. V, 5.	See note to Rev. line 5.
Rev. V,16.	*3*,6,24: should be 2,6,24.
Rev. VI, 3.	Reading uncertain.
Rev. VI, 5.	See note to Rev. line 5.

Commentary

I shall first comment on the text column by column, beginning briefly with the ones I found it necessary to compute for the sake of restoring the preserved columns, and then proceed to a few more general remarks. I shall adhere to the terminology of ACT as far as possible.

Column T, the date column, is reconstructed in the following manner. From well-preserved Λ-values in Column III, I found, via Table 3, the corresponding Φ-values, which turned out to be Φ_1-values (i. e., Φ-values associated with conjunction) and Column Φ of the text was thus recaptured. Assuming that these Φ-values are connectible to those of the ACT System A texts, the date column could then be provisionally restored. Assuming further that the solar longitudes, too, are connectible to those in the ACT corpus, the dates then yielded the solar longitudes given in Column B. These solar longitudes were later confirmed by Column Y (particularly Rev. IV, 11), as well as by columns which depend on length of daylight which, in turn, derives from solar longitude.

The reconstructed dates are then beyond doubt, though they are very early. The hitherto earliest known lunar ephemeris according to System A was ACT No. 1 (from Uruk) for the years S.E. 124–125.

To proceed with the reconstructed columns, Column C gives the length of daylight in large hours and is derived from Column B according to the standard System A scheme given in ACT.[12]

Column G, which played a large rôle in the introductory discussions, is derived from Column Φ according to the standard ACT set of rules.[13]

Column J, the correction for solar anomaly, is derived from Column B by the rules discussed above, and Column C' derives from Column C by the relation:

$$C_n' = \tfrac{1}{2}(C_{n-1} - C_n),$$

as in ACT.[14]

[12] ACT I, p. 47.
[13] ACT I, p. 60.
[14] ACT I, p. 62.

The first preserved column of our text is Column K, where

$$\mathrm{K}_n = \mathrm{G}_n + \mathrm{J}_n + \mathrm{C}_n',$$

i. e., it denotes the length of the month, but for 29^d, with a correction, C', built in to account for the monthly variation in the length of daylight; this is necessitated by the desire to denote the moment of conjunction relative to sunset. Column K is, as usual, abbreviated to three places.

Column II is Column M which gives the moment of conjunction; thus, Obv. II,1 says, that in S.E. –8, month XII, the conjunction of sun and moon happened on the 29th day, $5;29,46^{\mathrm{H}}$ before sunset (šú, short for *ana* šú *šamaš*, means *until sunset*). In order to compute the date of conjunction, one must know whether the previous month was full or hollow. This information would have been given by Column P, but our text is not preserved that far; thus I have made no attempt at restoring the dates of the conjunction, since I chose to work only with internal evidence. However, the dates can be quite securely reconstructed from Parker and Dubberstein's Chronology,[15] if one wishes.

The hours of conjunction are readily computed from K by the rule that

$$\mathrm{M}_n = \mathrm{M}_{n-1} - \mathrm{K}_n,$$

letting M stand for the hours. K is subtracted because M denotes the hours *before* sunset. M, too, is limited to three places; thus, my restoration of K and M may occasionally be off by one in the last digit.

Thus far our text has followed the pattern of System A lunar texts in ACT. The next columns are found here for the first time in an ephemeris, though Λ was known from procedure texts.

Column III gives Λ, month by month. In analogy with the situation for G, it is convenient to introduce a pure zig-zag function, $\hat{\Lambda}$, which has the same period as Λ and Φ (ultimately the anomalistic month) and which agrees with Λ on its linear stretches.

The parameters for $\hat{\Lambda}$ are

$$\begin{aligned} M\hat{\Lambda} &= 5;3,33,53,33,53,20^{\mathrm{H}} \\ m\hat{\Lambda} &= -0;46,18,8,53,20^{\mathrm{H}} \\ d\hat{\Lambda} &= 0;50,10,51,51,6,40^{\mathrm{H/m}} \end{aligned}$$

with reflexion parameters

[15] Richard A. Parker and Waldo H. Dubberstein, *Babylonian Chronology 626 B.C. – A.D. 75.* Providence, R.I., 1956.

Table 3.

Φ	Φ	Λ	Interpol.
2,13,20 ↑	2,13,20 ↑	3,55,33,20	0
2,15,27, 2,13,20 ↑	2,13, 2,13,20 ↑	3,55,33,20	1
2,15,44,48,53,20 ↑	2,12,44,26,40 ↑	3,55,15,33,20	2
2,16, 2,35,33,20 ↑	2,12,26,40 ↑	3,54,40	3
2,16,20,22,13,20 ↑	2,12, 8,53,20 ↑	3,53,46,40	4
2,16,38, 8,53,20 ↑	2,11,51, 6,40 ↑	3,52,35,33,20	5
2,16,55,55,33,20 ↑	2,11,33,20 ↑	3,51, 6,40	6
2,16,55,55,33,20 ↓	2,11,15,33,20 ↑	3,49,20	7
2,16,38, 8,53,20 ↓	2,10,57,46,40 ↑	3,47,15,33,20	8
2,16,20,22,13,20 ↓	2,10,40 ↑	3,44,53,20	9
2,16, 2,35,33,20 ↓	2,10,22,13,20 ↑	3,42,13,20	10
2,15,44,48,53,20 ↓	2,10, 4,26,40 ↑	3,39,15,33,20	11
2,15,27, 2,13,20 ↓	2, 9,46,40 ↑	3,36	12
2,15, 9,15,33,20 ↓	2, 9,28,53,20 ↑	3,32,26,40	13
2,14,51,28,53,20 ↓	2, 9,11, 6,40 ↑	3,28,35,33,20	14
2,14,33,42,13,20 ↓	2, 8,53,20 ↑	3,24,26,40	15
2,14,15,55,33,20 ↓	2, 8,35,33,20 ↑	3,20	16
2,13,58, 8,53,20 ↓	2, 8,17,46,40 ↑	3,15,15,33,20	17
2,13,40,22,13,20 ↓	2, 8 ↑	3,10,13,20	18
2,13,22,35,33,20 ↓	2, 7,42,13,20 ↑	3, 4,53,20	18; 8,45
2,13, 4,48,53,20 ↓	2, 7,24,26,40 ↑	2,59,30,44,26,40	18; 8,45
⋮	⋮	⋮	
2, 4,11,28,53,20 ↓	1,58,31, 6,40 ↑	18,12,57,46,40	18; 8,45
2, 3,53,42,13,20 ↓	1,58,13,20 ↑	12,50,22,13,20	17; 8,45
2, 3,35,55,33,20 ↓	1,57,55,33,20 ↑	7,45,33,20	16; 8,45
2, 3,18, 8,53,20 ↓	1,57,58, 8,53,20 ↓	2,58,31, 6,40	15; 8,45
2, 3, 0,22,13,20 ↓	1,58,15,55,33,20 ↓	− 1,30,44,26,40	14; 8,45
2, 2,42,35,33,20 ↓	1,58,33,42,13,20 ↓	− 5,42,13,20	13
2, 2,24,48,53,20 ↓	1,58,51,28,53,20 ↓	− 9,33,20	11
2, 2, 7, 2,13,20 ↓	1,59, 9,15,33,20 ↓	− 12,48,53,20	9
2, 1,49,15,33,20 ↓	1,59,27, 2,13,20 ↓	− 15,28,53,20	7
2, 1,31,28,53,20 ↓	1,59,44,48,53,20 ↓	− 17,33,20	5
2, 1,13,42,13,20 ↓	2, 0, 2,35,33,20 ↓	− 19, 2,13,20	3
2, 0,55,55,33,20 ↓	2, 0,20,22,13,20 ↓	− 19,55,33,20	1
2, 0,38, 8,53,20 ↓	2, 0,38, 8,53,20 ↓	− 20,13,20	

$$2M - d = 9;16,56,55,16,40$$
$$2m + d = -0;42,25,25,55,33,20$$

and the period

$$P_{\hat{\Lambda}} = P_{\Lambda} = P_{\Phi} = \frac{1,44,7}{7,28}.$$

Λ agrees with $\hat{\Lambda}$ for

$$0;12,50,22,13,20 \leqq \Lambda \leqq 3;4,53,20.$$

Beyond these limits, Λ is derived from Φ by the scheme given in Table 3. In this table the arrow after a Φ-value indicates whether it belongs to an ascending or descending branch of the zig-zag function; the last column presents interpolation coefficients referring to the subsequent interval.

Column IV gives the function Y, the correction to Λ for solar anomaly, which has been discussed above. As said, the Y-values, but for the first four, are erroneously denoted "lal" by a scribe who was probably familiar with the similarly appearing Column J. However, as we shall see, the values in Column $\tilde{\mathrm{K}}$ show clearly that Y is to be taken as positive when not zero.

Transitional values are rare, as I said above. The one in Rev. IV, 11, i. e.

$$\mathrm{Y} = 0;7,13,40,30^{\mathrm{H}}$$

follows precisely the rules set out in the introduction. It would, by itself, have sufficed as a base for reconstructing the solar longitudes, had I only understood Column Y in time.

Column V, which I call $\tilde{\mathrm{C}}$', is a correction to Λ for the change in length of daylight, analogous to the correction C' to G.

It is precisely derived from C by the rule

$$\tilde{\mathrm{C}}'_n = \tfrac{1}{2}(\mathrm{C}_{n-12} - \mathrm{C}_n).$$ [16]

In order to restore the first 12 values of this column, I have provided the solar longitudes and the corresponding values of C in Table 4.

Column VI, the last preserved column of the tablet, of which but little remains, I call $\tilde{\mathrm{K}}$. It is in an analogous relation to Λ as K is to G, for

$$\tilde{\mathrm{K}}_n = \Lambda_n + \mathrm{Y}_n + \tilde{\mathrm{C}}'_n$$

abbreviated to three places.

[16] Cf. ACT No. 200b, Section 3, which gives the change in C for 12 month intervals. If one halves these values, one gets $\tilde{\mathrm{C}}'$ under certain conditions.

TABLE 4.

T	B_1	C_1
S.E. −9, XII_2	♈ 21;48,45	3, 7,52,30
−8, I	♉ 19;56,15	3,23,58,30
II	♊ 18; 3,45	3,33, 4,30
III	♋ 16;11,15	3,35,40,30
IV	♌ 14;18,45	3,30, 9,30
V	♍ 12;26,15	3,18,22,30
VI	♎ 12;24	2,58,24
VII	♏ 12;24	2,39, 2,24
VIII	♐ 12;24	2,27,40,40
IX	♑ 12;24	2,24,19,12
X	♒ 12;24	2,28,57,36
XI	♓ 12;24	2,41,36
l.1 = XII	♈ 11;26,15	3, 0,57,30

The few numbers which remain of $\tilde{K}$ suffice to show that the designation of the Y-values as negative, but for the first four, is an error without consequence.

The discovery of the columns $\tilde{C}$' and $\tilde{K}$ makes me unable to see any justification for Λ except that it, or rather $\tilde{K}$, provides a much needed check for Column M. Using the relations of G to Λ, J to Y, and C to C' and $\tilde{C}$', it is readily seen that we have, at least ideally,

$$M_{n-12} - \tilde{K}_n = M_n \pmod{6^H};$$

to give a specific example from the text:

$$\begin{array}{rcr} M_{15} & = & 3;57,39 \\ -\tilde{K}_{27} & = & -1;56,33 \\ \hline & & 2;\ 1,\ 6 \end{array}$$

and the text gives:

$$M_{27} = 2;1,2.$$

The slight disagreement was, in part, to be expected from what we found in the above comparison between G and Λ; the relations between J and Y, and C' and $\tilde{C}$' are, of course, exact.

Since M is a conglomerate of quite unrelated parts, it has, in modern times, always been a problem to check M-values, and the problem is, of course, aggravated by the very nature of M which preserves an error once introduced, as well as by M's importance. It is, then, not very surprising to learn that the Babylonians had constructed $\tilde{K}$, and its ingredients, as a checking device, of necessity elaborate, nor is it, then, odd that Λ and the other new functions do not appear elsewhere in the regular, finished ephemerides.

If $\tilde{\mathrm{K}}$ is to serve well as a control on M, it is very desirable that its constituent parts be as independent of their analogues in K as possible. As we have seen, Λ is found directly from Φ and independently of G; so I am confident that we shall eventually find textual evidence for the rules I have given above for deriving Y directly from the solar longitude without J as an intermediary.

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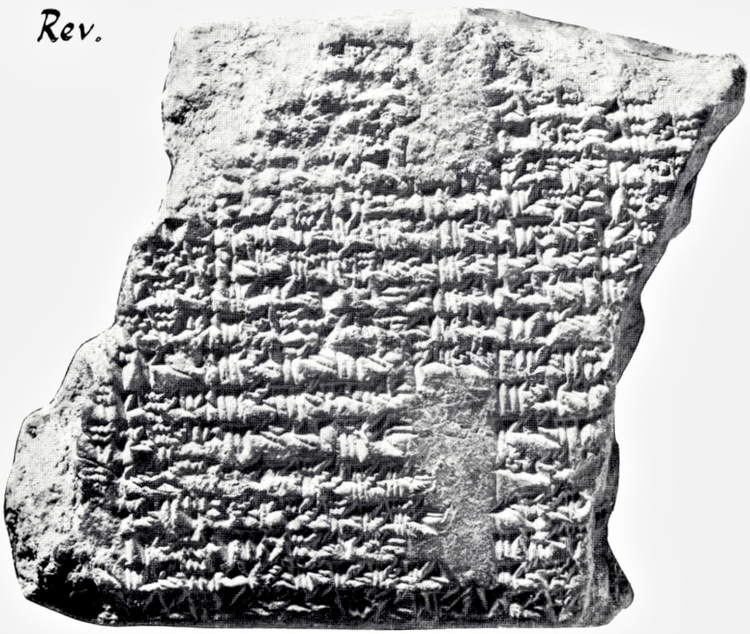

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